4. I. I. Lishtvan and N. G. Korol', Basic Properties of Peat and Methods of Their Determination [in Russian], Nauka i Tekhnika, Minsk (1975).
5. G. P. Korchagin and S. L. Koreneva, Compression Measurement and Rotational Shear in Engineering Geology [in Russian], Nedra, Moscow (1976).
6. G. M. Lyakhov, Principles of the Dynamics of Explosion in Soils and Liquids [in Russian], Nedra, Moscow (1964).
7. G. M. Lyakhov, Waves in Soils and Porous Multicomponent Media [in Russian], Nauka, Moscow (1982).

APPLICATION OF A GENERALIZED FORMULATION OF THE STEFAN PROBLEM TO INVESTIGATION OF RADIATION-CONDUCTIVE HEAT TRANSFER

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Study of the influence of internal thermal radiation on the temperature distribution formation in semitransparent materials by using the classical Stefan model, assuming the presence of a plane interface between the liquid and solid phases, showed that the monotonic nature of the temperature distribution is spoiled ahead of the plane front. This fact was explained in [1, 2] as heating of the solid phase during melting and over cooling of the liquid during solidification caused by heat transfer due to radiation. Overheating and overcooling are metastable states of a substance, but crystal overheating in the domain bounding the liquid phase is not generally realized [3]. Dendritic growth is detected before the crystallization front upon the appearance of an overcooled zone. Moreover, an independent volume generation of crystals is possible [4]. Therefore, instead of overheating and overcooling, a two-phase or transition zone appears in the semitransparent medium, in which partial melting or solidification occurs, caused by solid-phase absorption of thermal radiation or because of radiation cooling. Also confirmed in [5] is the reality of the appearance of a transition zone in a semitransparent medium. The spoilage of the classical Stefan condition and the appearance of a transition zone are also mentioned in [6, 7].

A generalized model, proposed in [5, 7], according to which the material under consideration consists of three sublayers (Fig. 1, zone 1: liquid, 2: solid, 3: two-phase), is used in this paper to investigate the influence of thermal radiation on the phase transformation process in a layer of semitransparent material. The initial layer temperature is below the melting point T_m; then the temperature of the left wall acquires a temperature $T_1 > T_m$ and is later maintained constant. At the initial time, surface melting predominates because of the high temperature gradient. After a certain time, a transition zone appears because of the rapid penetration of the thermal radiation into the solid phase. It is considered that the thermophysical and optical properties are constant in all the phases, a unique melting point exists, and a two-phase domain is in thermodynamic equilibrium at this temperature. The density change during melting is assumed insignificant; consequently, convective motion is neglected.

In a generalized formulation the Stefan problem reduces to determining the temperature as a continuous function $\Theta(x, t)$ satisfying the energy equation

$$\frac{\partial u}{\partial t} = \operatorname{div}(k\nabla\Theta) + f(\Theta); \tag{1}$$

$$u(\Theta) = \int_0^{\Theta} c(\xi)\,d\xi - \alpha(\Theta - \Theta_m)\lambda,\quad \alpha = \begin{cases} 0, & \Theta > \Theta_m, \\ [0, 1], & \Theta = \Theta_m, \\ 1, & \Theta < \Theta_m, \end{cases} \tag{2}$$

within the domain $\{0 < x < 1,\ 0 < t < T\}$, where $u(\Theta)$ is the enthalpy undergoing a discontinuity of the first kind and defined ambiguously for $\Theta = \Theta_m$, and λ is the latent heat of melting.

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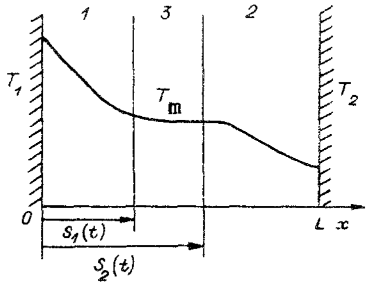

Fig. 1

Taking (1) and (2) into account, we obtain a system of equations (with dimensionless parameters)

$$C_1 \frac{\partial \Theta}{\partial t} = NK_1 \frac{\partial^2 \Theta}{\partial x^2} - \frac{1}{4} \frac{\partial q_1}{\partial x}, \quad 0 < x < S_1(t), \tag{3}$$

$$C_2 \frac{\partial \Theta}{\partial t} = NK_2 \frac{\partial^2 \Theta}{\partial x^2} - \frac{1}{4} \frac{\partial q_2}{\partial x}, \quad S_2(t) < x < 1,$$

$$\frac{\partial \alpha}{\partial t} = \frac{1}{4\lambda} \frac{\partial q_3}{\partial x}, \quad \Theta = \Theta_m, \quad S_1(t) \leqslant x \leqslant S_2(t).$$

Here $C_i = c_i/c_r$; $K_i = k_i/k_r$; $x = x/L$; $\Theta = T/T_r$; $\lambda = \lambda/c_rT_r$; $t = 4\sigma_0 T_r^3 t/c_rL$; $q_i = E_i/\sigma_0 T_r^4$; $S_i = S_i/L$; $N = k_r/4\sigma_0 LT_r^3$ is the conductive-radiation parameter, σ_0 is the Stefan-Boltzmann constant, r is the index of the characteristic parameter, and $\alpha(x, t)$ determines the fraction of the solid phase in the transition zone.

The initial conditions are

$$S_1(0) = S_2(0) = S_0, \quad \Theta(x, 0) = \Theta_0(x); \tag{4}$$

the boundary conditions are

$$\Theta(0, t) = \Theta_1, \quad \Theta(1, t) = \Theta_2. \tag{5}$$

The conditions on the inner boundaries $S_1(t)$ and $S_2(t)$ are found from the equations on the strong discontinuity

$$\alpha|_{S_1^+} \lambda \frac{dS_1}{dt} = -NK_1 \frac{\partial \Theta}{\partial x}\Big|_{S_1^-}; \tag{6}$$

$$\left(1 - \alpha|_{S_2^-}\right) \lambda \frac{dS_2}{dt} = NK_2 \frac{\partial \Theta}{\partial x}\Big|_{S_2^+}. \tag{7}$$

Since $dS_2/dt \geq 0$, $\partial\Theta/\partial x|_{S_2^+} < 0$, then the relationship (7) is satisfied for $\alpha|_{S_2^-} = 1$, $\partial\Theta/\partial x|_{S_2^+} = 0$.

The medium is gray, absorbing, and radiating; the refractive indices of all the layers are $n_1 = n_2 = n_3 = 1.5$, whereupon scattering is neglected. The absorption coefficient of the transition zone is $\varkappa_3 = (\varkappa_1 + \varkappa_2)/2$. The specimen boundary surfaces are black ($\varepsilon_1 = \varepsilon_2 = 1$).

The resultant radiation fluxes that enter into (3) are found from the formal solutions of the radiation transport equations for each layer

$$q_1(x, t) = 2\pi \left\{ \int_0^1 \left[I^+(0) \exp(-h_1 x/\mu_1) + \int_0^x h_1 n_1^2 \frac{B(x')}{\mu_1} \exp(-h_1(x - x')/\mu_1)\, dx' - \right.\right. \tag{8}$$

$$- I^-(1) \exp(-h_2(1 - S_2)/\mu_2 - h_3(S_2 - S_1)/\mu_3 - h_1(S_1 - x)/\mu_1) -$$

$$- \int_{S_2}^1 h_2 n_2^2 \frac{B(x')}{\mu_2} \exp(-h_2(x' - S_2)/\mu_2 -$$

$$- h_1(S_1 - x)/\mu_1 - h_3(S_2 - S_1)/\mu_3)\, dx' -$$

$$-\int_{S_1}^{S_2} h_3 n_3^2 \frac{B(x')}{\mu_3} \exp(-h_3(x'-S_1)/\mu_3 -$$

$$-h_1(S_1-x)/\mu_1)\,dx' - \int_x^1 h_1 n_1^2 \frac{B(x')}{\mu_1} \times$$

$$\left.\left.\times \exp(-h_1(x'-x)/\mu_1)\,dx'\right]\mu_1 d\mu_1\right\},\ 0<x<S_1(t),$$

$$q_2(x,\ t) = 2\pi\left\{\int_0^1\left[I^+(0)\exp(-h_1S_1/\mu_1 - h_3(S_2-S_1)/\mu_3 -\right.\right.$$

$$-h_2(x-S_2)/\mu_2) + \int_0^{S_1} h_1 n_1^2 \frac{B(x')}{\mu_1} \exp(-h_1(S_1-x')/\mu_1 - h_2(x-S_2)/\mu_2 -$$

$$-h_3(S_2-S_1)/\mu_3)\,dx' + \int_{S_1}^{S_2} h_3 n_3^2 \frac{B(x')}{\mu_3} \exp(-h_3(S_2-x')/\mu_3 -$$

$$-h_2(x-S_2)/\mu_2)\,dx' + \int_{S_2}^{x} h_2 n_2^2 \frac{B(x')}{\mu_2} \exp(-h_2(x-x')/\mu_2)\,dx' -$$

$$-I^-(1)\exp(-h_2(1-x)/\mu_2) - \int_x^1 h_2 n_2^2 \frac{B(x')}{\mu_2} \times$$

$$\left.\left.\times \exp(-h_2(x'-x)/\mu_2)\,dx'\right]\mu_2 d\mu_2\right\},\ S_2(t)<x<1,$$

$$q_3(x,\ t) = 2\pi\left\{\int_0^1\left[I^+(0)\exp(-h_1S_1/\mu_1 - h_3(x-S_1)/\mu_3) +\right.\right.$$

$$+\int_0^{S_1} h_1 n_1^2 \frac{B(x')}{\mu_1} \exp(-h_1(S_1-x')/\mu_1 - h_3(x-S_1)/\mu_3)\,dx' +$$

$$+\int_{S_1}^{x} h_3 n_3^2 \frac{B(x')}{\mu_3} \exp(-h_3(x-x')/\mu_3)\,dx' - I^-(1)\exp(-h_2(1-S_2)/\mu_2 -$$

$$-h_3(S_2-x)/\mu_3) - \int_{S_2}^{1} h_2 n_2^2 \frac{B(x')}{\mu_2} \exp(-h_2(x'-S_2)/\mu_2 - h_3(S_2-x)/\mu_3)\,dx' -$$

$$\left.\left.-\int_x^{S_2} h_3 n_3^2 \frac{B(x')}{\mu_3} \exp(-h_3(x'-x)/\mu_3)\,dx'\right]\mu_3 d\mu_3\right\},\ S_1(t)\leqslant x\leqslant S_2(t).$$

Here $h_i = \varkappa_i L$ is the optical thickness of the layers, $B(x')$ is the radiation intensity of an absolutely black body in a vacuum, μ_i is the cosine of the angle between the radiation direction and the x-axis, and $I^+(0) = \varepsilon_1 n_1^2 B(0)$, $I^-(1) = \varepsilon_2 n_2^2 B(1)$ are the boundary radiation intensities.

An integro-interpolation method [8] is used for the numerical solution of the boundary value problem (3)-(7), for which integral conservation laws are satisfied for the difference approximation of the heat-conduction equations, the Newton method is used to determine $S_1(t)$ and $S_2(t)$, and $d\alpha/dt$ is approximated by a finite-difference ratio. Integrals of the form $K_n(h) = \int_0^1 \mu^{n-2}\exp(-h/\mu)\,d\mu$ were evaluated by Gauss quadratures, and those of the form $I(x) = \int_0^x f(x')K_n(-\varkappa(x-x'))\,dx'$ by trapezoid formulas.

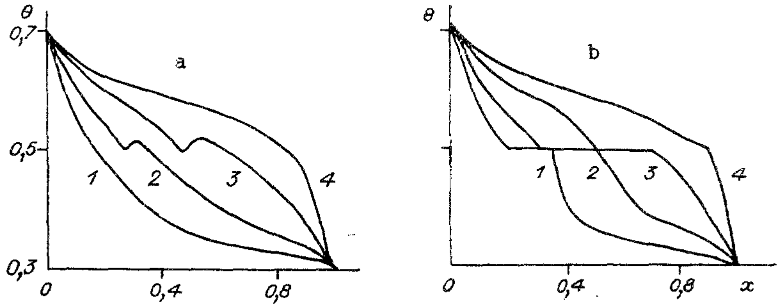

Fig. 2

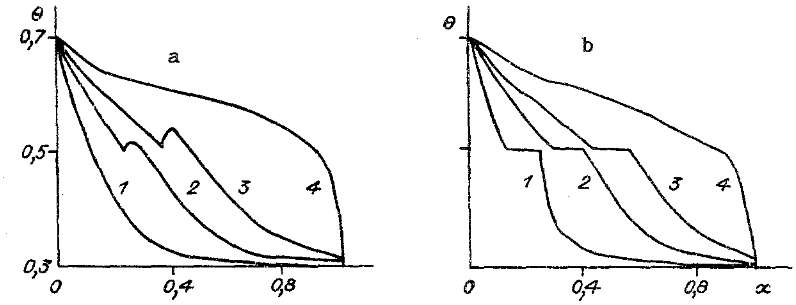

Fig. 3

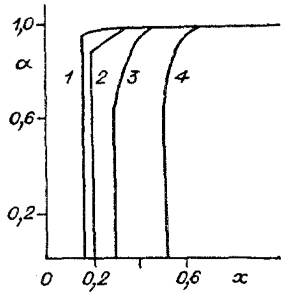

Fig. 4

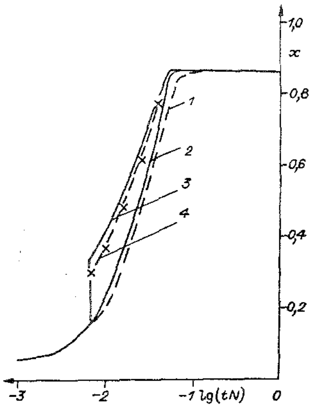

Fig. 5

Computations were performed for the following values of the dimensionless parameters: $C_1 = 0.75$, $C_2 = 1$, $K_1 = 2$, $K_2 = 1$, $\lambda = 0.1$, which correspond approximately to the melting of fluorite ($T_m = 1700$ K, $k_r = 9$ W/(m·K), L = 10 cm) [1]. The boundary temperatures are $\Theta_1 = 0.7$, $\Theta_2 = \Theta_0 = 0.3$. The melting point is $\Theta_m = 0.5$, N = 0.01. According to the classical model of a phase transition, the computations were performed until a monotonic temperature profile was retained, then they were continued by the generalized model.

Figures 2a and 2b show temperature profiles in a specimen obtained in a computation by the classical and generalized models. The optical thicknesses of the layers are $h_1 = \varkappa_1 L = 1$, $h_2 = \varkappa_2 L = 2$, $h_3 = \varkappa_3 L = (h_1 + h_2)/2$. As the time increases the two-phase layer thickness diminishes (curve 1 is $\Delta s^{(1)} = S_2 - S_1 = 0.1794$, 2 is $\Delta s^{(2)} = 0.1754$, 3 is $\Delta s^{(3)} = 0.14$). The two-phase layer vanishes upon emergence in the stationary temperature regime. If we take $h_3 = 2$, then the transition zone thickness becomes less than for $h_3 = 1.5$ ($\Delta s^{(1)} = 0.1465$, $\Delta s^{(2)} = 0.1313$, $\Delta s^{(3)} = 0.106$) since the thermal radiation from the hotter liquid phase is absorbed more strongly by the layer lying nearest to the liquid phase.

Presented in Figs. 3a and b are the temperature distributions computed by the classical and generalized models for $h_1 = 1$, $h_2 = 5$, $h_3 = 3$. As should have been expected, the transition zone dimensions are still less than in Fig. 2b (curve 1 is $\Delta s^{(1)} = 0.1126$, 2 is $\Delta s^{(2)} = 0.107$, 3 is $\Delta s^{(3)} = 0.101$).

Shown in Fig. 4 is the distribution of the solid-phase fraction α(x, t) in the two-phase domain at different times (curve 1 is t = 0.662; 2 is 0.76; 3 is 1.27; 4 is 2.5) corresponding to the temperature profiles presented in Fig. 2b. As is seen, the fraction of the melted solid body part is diminished with the depth of the two-phase layer since the layers lying close to the hot liquid phase absorb more radiation energy; but only the part of the solid body forming the two-phase layer melts because the quantity of absorbed energy does not equal the latent heat of melting of the solid body. Surface melting evidently occurs because of heat conduction, resulting in the formation of a pure liquid layer (α = 0), while internal melting resulting in non-zero values of α proceeds because of thermal radiation.

Motion of the transition zone boundaries is illustrated in Fig. 5, where curve 2 corresponds to $S_1(t)$, 3 to $S_2(t)$ for $h_3 = 1.5$, 4 for $h_3 = 2$, while curve 1 is motion of the front computed by the classical phase transition model. It is seen that extraction of the two-phase intermediate domain accelerates the melting process.

Spoilage of the monotoneity [9] is not observed in the temperature distributions in computations by the classical phase transition model for N ≥ 0.05, which means that the contribution from thermal radiation to heat transfer for these values is insufficient for the appearance of a two-phase domain. In such cases the classical phase transition model, which becomes unacceptable for the appearance of the transition zone being taken into account in the generalized model, is perfectly suitable. As soon as the transition zone vanishes, the generalized model again goes over into the classical model.

The problem formulation considered is somewhat idealized since the optical properties of the transition zone have not been studied theoretically or experimentally, and the structure of the two-phase domain being formed here is unknown.

LITERATURE CITED

1. M. Abrams and R. Viskanta, "The effects of radiative heat transfer upon the melting and solidification of semitransparent crystals," J. Heat Transfer, 96, No. 2 (1974).
2. N. A. Savvinova, "Phase transitions in a plane layer with radiation taken into account," Molecular Physics of Nonequilibrium Systems [in Russian], Novosibirsk (1984).
3. A. R. Ubbelode, Melted State of Substances [in Russian], Metallurgiya, Moscow (1982).
4. B. Ya. Lyubov, Theory of Crystallization in Large Volumes [in Russian], Nauka, Moscow (1975).
5. S. H. Chan, D. H. Cho, and G. Kosamustafaogullari, "Melting and solidification with internal radiative transfer – a generalized phase change model," Int. J. Heat Mass Transfer, 26, No. 4 (1983).
6. A. M. Meirmanov, "Example of the non-existence of a classical solution of the Stefan problem," Dokl. Akad. Nauk SSSR, 258, No. 3 (1981).
7. A. M. Meirmanov, Stefan Problem [in Russian], Nauka, Novosibirsk (1986).
8. A. A. Samarskii, Theory of Difference Schemes [in Russian], Nauka, Moscow (1977).
9. A. L. Burka, N. A. Rubtsov, and N. A. Savvinova, "Nonstationary radiation-conduction heat transfer in a semitransparent medium with a phase transition," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1987).